Introduction to Differentiable Manifolds

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Exercise series 7 **2021–11–09**

Exercise 7.1 (Trivial vector bundles)**.** .

(a) Show that a vector bundle is trivial if and only if it has a global frame.
(b) Show that the vector bundle TS^1 is trivial.

Exercise 7.2 (Properties of smooth vector fields)**.** Let M be a smooth manifold and let $X : M \to TM$ be a vector field. Show that the following are equivalent:

(a) X is a smooth vector field.
(b) The component functions of X are smooth with respect to all charts of one particular smooth atlas of M.
(c) For any smooth function $f : U \to \mathbb{R}$ on an open set $U \subset M$, the function $Xf : U \to \mathbb{R}$ defined by $Xf(p) := X_p(f)$ is smooth.

Exercise 7.3 (Vector field on S^2)**. Optional.**
Show that there is a smooth vector field on S^2 which vanishes at exactly one point. (*Hint:* Try using stereographic projection and consider one of the coordinate vector fields.)