Exercise 12.1. Show that the torus $\mathbb{T}^n$ is orientable.

Exercise 12.2. Show that a product $M_0 \times \cdots \times M_{k-1}$ of several oriented manifolds is naturally oriented.

Exercise 12.3. Let M be a differentiable n-manifold. Show that the sign $\operatorname{sgn} \omega$ of a nonvanishing n-form on M, defined by

$$(\operatorname{sgn} \omega)|_p(X^0, \ldots, X^{n-1}) := \operatorname{sgn}(\omega|_p(X^0, \ldots, X^{n-1}))$$

for $p \in M$ and $X^0, \ldots, X^{n-1} \in \mathrm{T}_p M$, is an orientation on M, and every orientation on M is the sign of some nonvanishing n-form.

Exercise 12.4 (Projective plane)**.** Show that the projective plane $\mathbb{P}^2$ is not orientable.

Exercise 12.5. Show that the Mobius band is not orientable.

1. Manifolds with boundary

Exercise 12.6. Let M be an n-dimensional $\mathcal{C}^r$ manifold with boundary.

(a) Show that Int M and ∂M are disjoint sets.
(b) Show that ∂M, endowed with the subspace topology, can be given the structure of an $n-1$-dimensional $\mathcal{C}^r$ manifold (without boundary) such that the inclusion map into M is $\mathcal{C}^r$.